

12. Prove that the poles of tangents to the parabola  $y^2 = 4ax$  with respect to the parabola  $y^2 = 4bx$  lie on a parabola.
13. One focus of a hyperbola is located at the point  $(1, -3)$  and the corresponding directrix is the line  $y = 2$ . Find the equation of the hyperbola if its eccentricity is  $3/2$ .
14. If  $PSQ$  is a chord passing through the focus  $S$  of a conic and  $l$  is semi latus rectum, show that  $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$ .
15. Evaluate  $\int \frac{1}{(1-x)(4+x^2)} dx$ .
16. Solve  $(x^2 - y^2) dx - xy dy = 0$ .
17. Solve  $(1 + y^2) dx = (\tan^{-1} y - x) dy$ .

### SECTION - C

5 × 7 = 35

### LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. Find the equation of the circle whose centre lies on  $X$ -axis and passing through  $(-2, 3)$  and  $(4, 5)$ .
19. Find the coordinates of the limiting points of the coaxial system determined by  $x^2 + y^2 + 2x - 6y = 0$  and  $2x^2 + 2y^2 - 10y + 5 = 0$ .
20. Find the eccentricity, coordinates of foci, length of latus rectum and equations of directrices of the ellipse  $9x^2 + 16y^2 - 36x + 32y - 92 = 0$ .
21. If  $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$  then show that  $(1+x^2)y_2 + 3xy_1 + y = 0$  and hence deduce that  $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0$ .
22. Obtain reduction formula for  $I_n = \int \sin^n x dx$  for an integer  $n \geq 2$  and deduce the value of  $\int \sin^4 x dx$ .
23. Show that  $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$ .
24. Dividing  $[0, 6]$  into 6 equal parts, evaluate  $\int_0^6 x^3 dx$  approximately by using :  
 i) Trapezoidal rule  
 ii) Simpson's rule